Week 12

TAM 210/211 Introduction of Statics

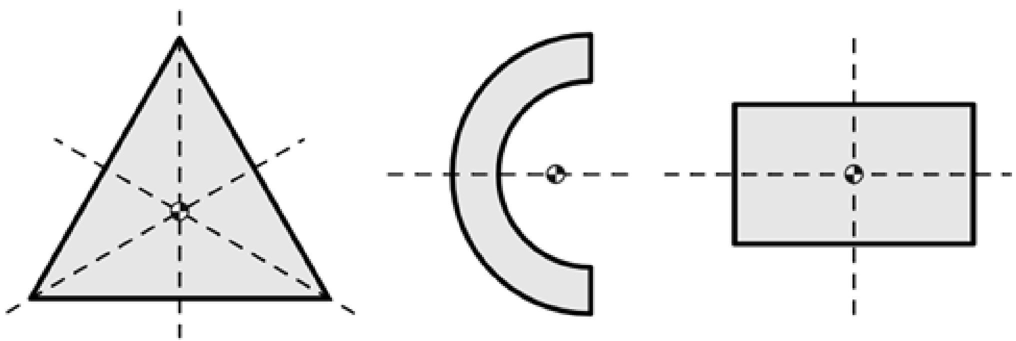
WEEK 12 – Center of Gravity (CoG)

- **Center of mass, center of gravity, and centroid**
  - Center of mass: average location of the mass
  - Center of gravity: average location of the weight
  - **Centroid:** geometric center (i.e., line, area, or volume)
  - **Centroid** = center of mass = center of gravity (if $\rho = const.$, $g = const.$ along an object)

	Center of mass	Center of gravity	Centroid
Equation (integral form)			
Equation (summation form)			

WEEK 12 – Center of Gravity (CoG)

- **How to find the centroid of an object – integral method**
  - Define the coordinate system
  - Define the infinitesimally small element (usually dL, dA, or dV)
  - Express the element in terms of the defined coordinate system
  - Define the centroid $(\tilde{x}, \tilde{y})$ of the element
    - Centroids of the basic shapes: https://www.mechref.org/sta/geometric_properties/?origin=coursemenu (reference pages)
    - https://engineeringstatics.org/Chapter_07-centroids.html (online textbook)
    - Identify any symmetry: the centroid is on the line of symmetry



(Reference: online textbook https://engineeringstatics.org/Chapter_07-centroids.html)

  - Use the integral form of equation to solve for the centroid

i.e., $\bar{x} = \dfrac{\int \tilde{x} dA}{\int dA}$, $\quad \bar{y} = \dfrac{\int \tilde{y} dA}{\int dA}$

WEEK 12 – Center of Gravity (CoG)

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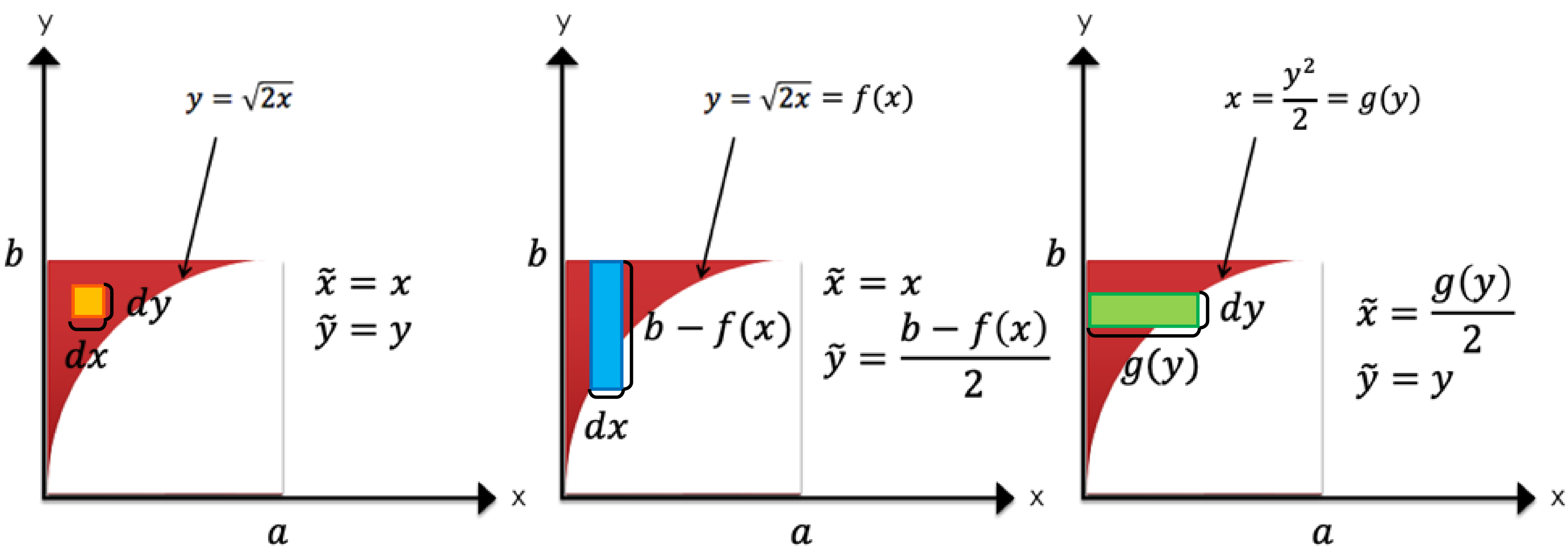
  - Express the element in terms of the defined coordinate system
  - Define the centroid $(\tilde{x}, \tilde{y})$ of the element
  - Use the integral form of equation to solve for the centroid (i.e., $\bar{x} = \frac{\int \tilde{x} dA}{\int dA}, \bar{y} = \frac{\int \tilde{y} dA}{\int dA}$)

* more details: https://engineeringstatics.org/centroids-by-integration.html (online textbook)

Homework #5A



WEEK 12 – Center of Gravity (CoG)

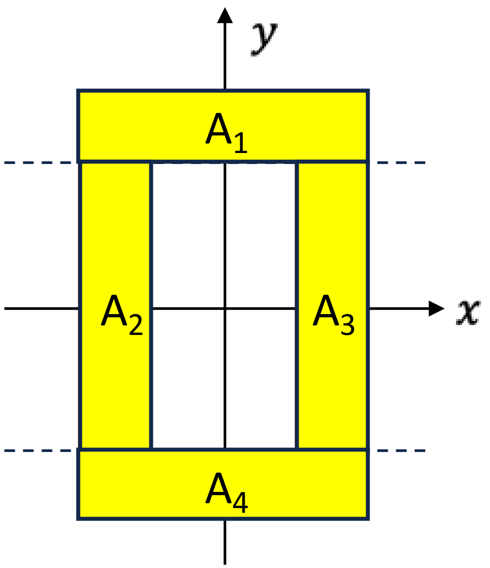
- **How to find the centroid of an object – summation method (example)**
  - Define the coordinate system
  - Break the shape into simpler basic shapes
  - Define the centroid $(\tilde{x}, \tilde{y})$ of the element
    - Centroids of the basic shapes: https://www.mechref.org/sta/geometric_properties/?origin=coursemenu (reference pages) https://engineeringstatics.org/Chapter_07-centroids.html (online textbook)
    - Identify any symmetry: the centroid is on the line of symmetry
  - Use the summation form of equation to solve for the centroid

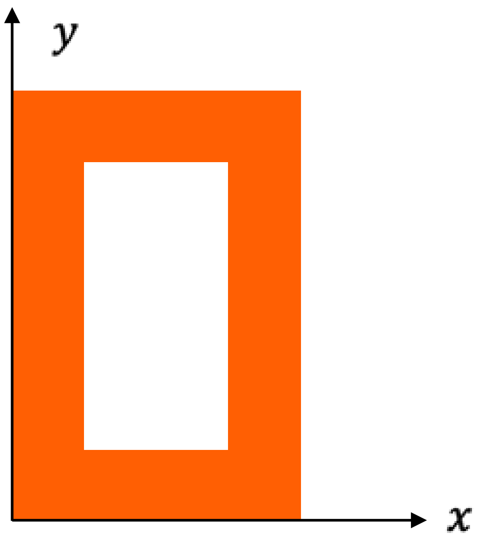
i.e., $\bar{x} = \dfrac{\sum \tilde{x}_i A_i}{\sum A_i}$

$$\bar{y} = \frac{\sum \tilde{y}_i A_i}{\sum A_i}$$

$$\bar{x} = \frac{\tilde{x}_1 A_1 + \tilde{x}_2 A_2 + \tilde{x}_3 A_3 + \tilde{x}_4 A_4}{A_1 + A_2 + A_3 + A_4}$$

$$\bar{y} = \frac{\tilde{y}_1 A_1 + \tilde{y}_2 A_2 + \tilde{y}_3 A_3 + \tilde{y}_4 A_4}{A_1 + A_2 + A_3 + A_4}$$





WEEK 12 – Center of Gravity (CoG)

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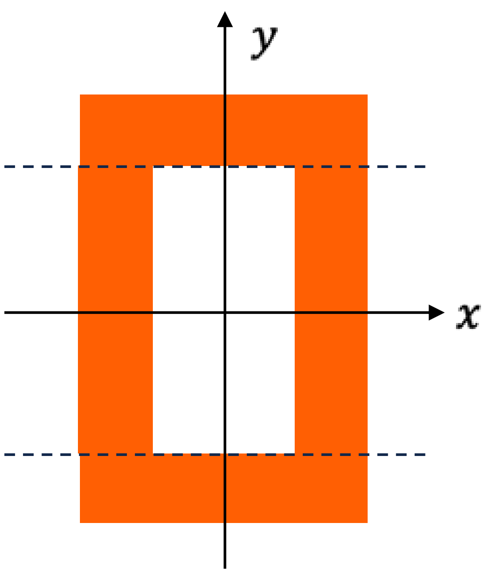
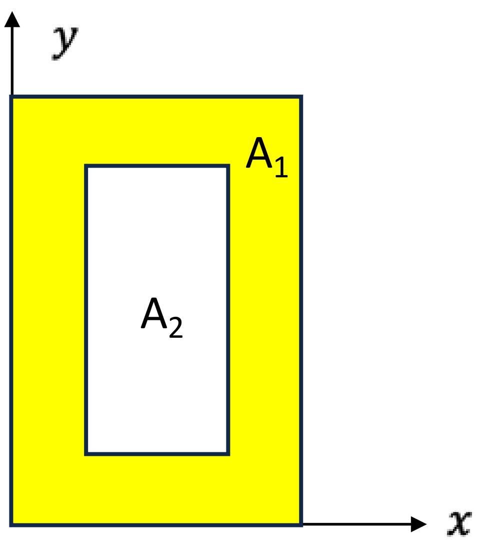
$$\bar{y} = \frac{\sum \tilde{y}_i A_i}{\sum A_i}$$

$$\bar{x} = \frac{\tilde{x}_1 A_1 - \tilde{x}_2 A_2}{A_1 - A_2}$$

$$\bar{y} = \frac{\tilde{y}_1 A_1 - \tilde{y}_2 A_2}{A_1 - A_2}$$



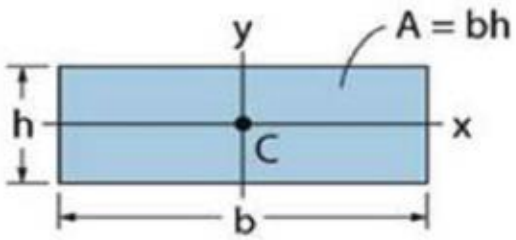


WEEK 12 – Center of Gravity (CoG)

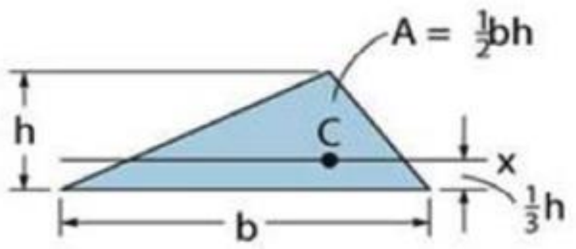
- **Additional instructions for Problem #3**
  - Figure 3 → figure on p.2
  - About arc (not sector) → dL (not dA)

Solve Problem #3

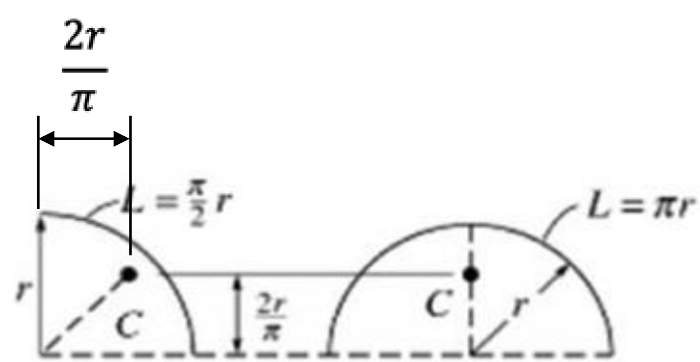
1. Use polar coordinates → r, θ
2. Use Cartesian coordinates → x, y



Rectangular area



Triangular area



Quarter and semicircle arcs